

# **Tetraquarks, molecular states from Lattice**

***Nilmani Mathur***

**Department of Theoretical Physics  
Tata Institute, India**

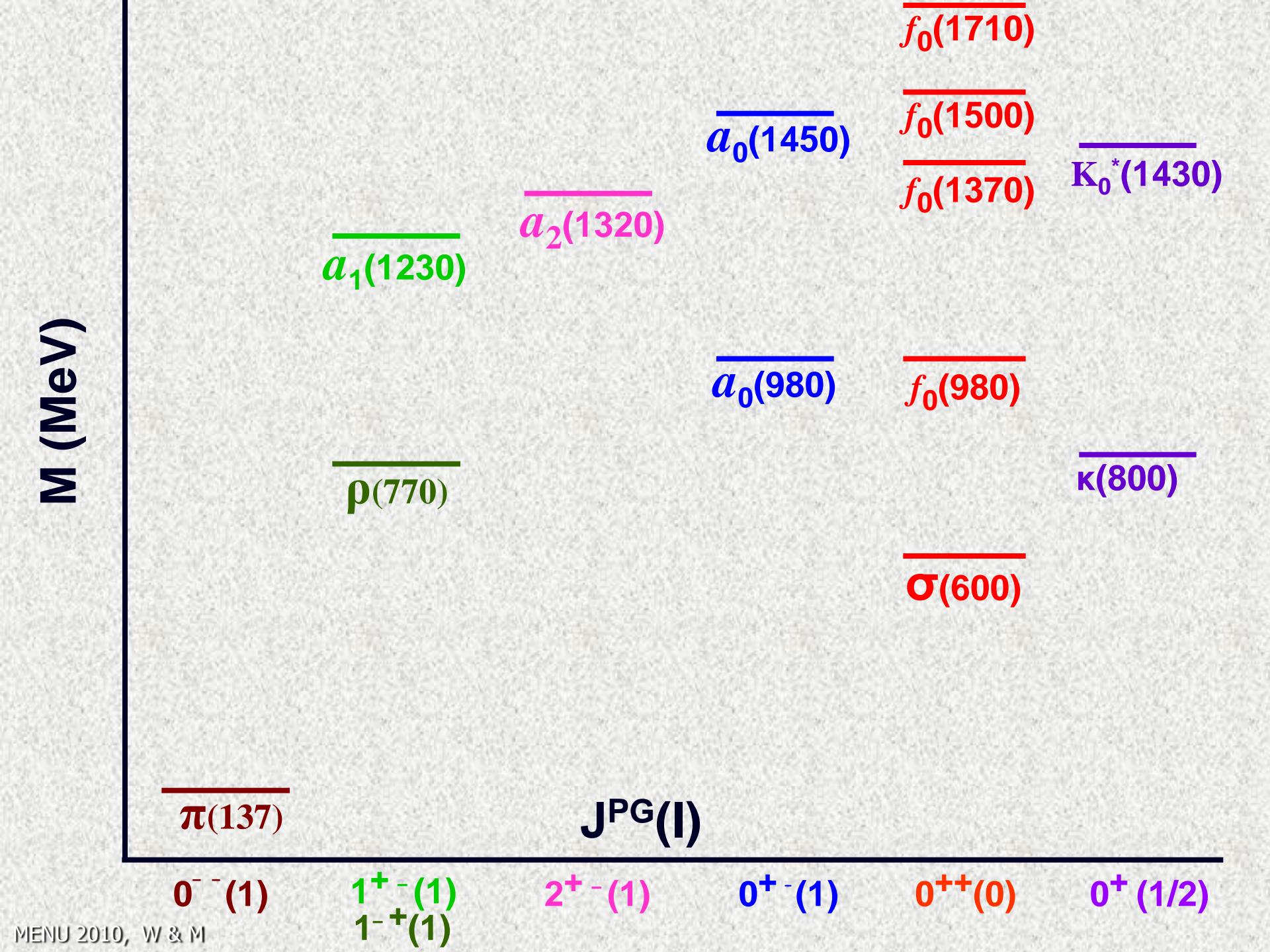
# Tetraquark states

## ➤ What is a tetraquark state?

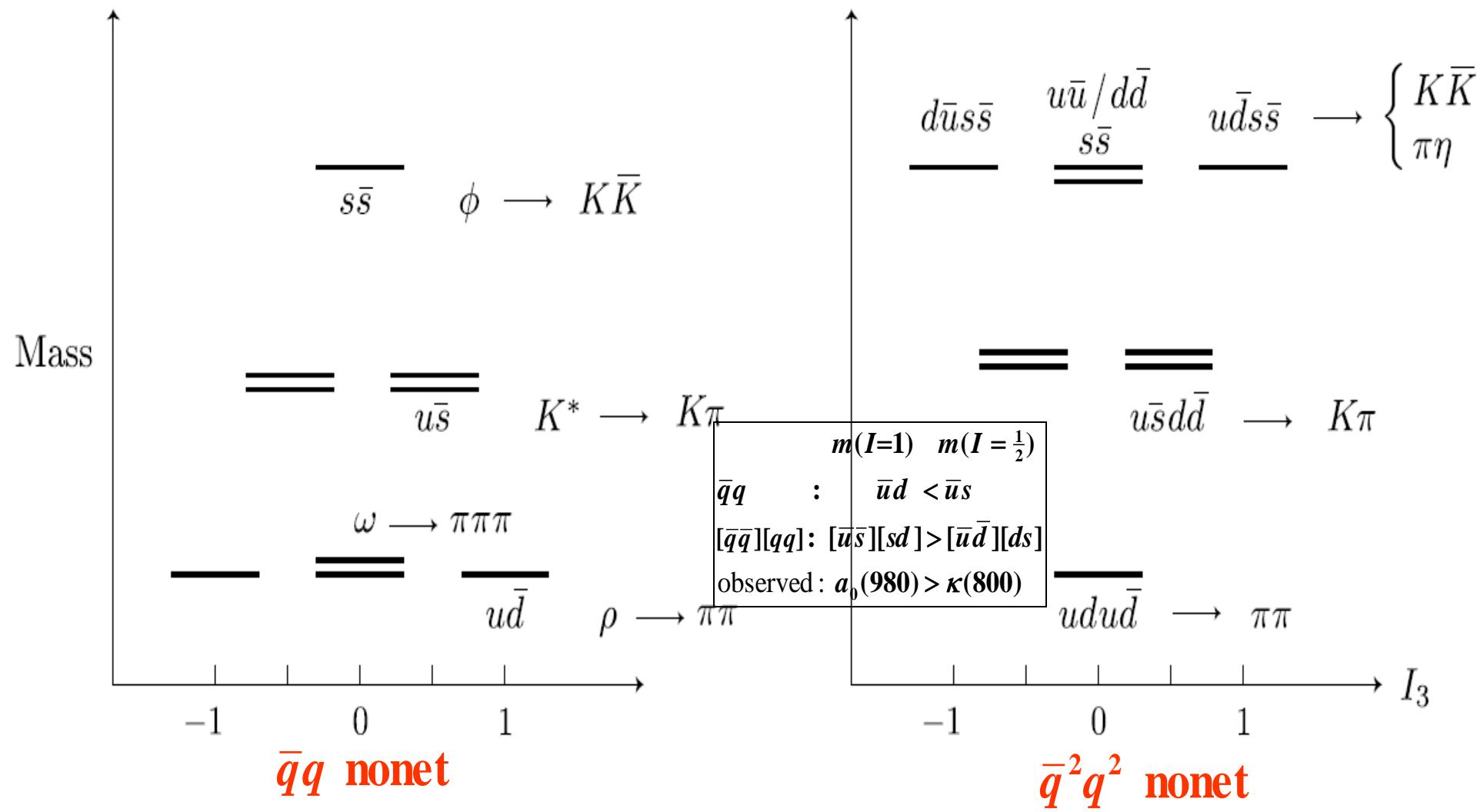
Large 4 quark component in Fock space in the same way mesons have large 2 quark and baryons have large 3 quark components.

## ➤ Why a tetraquark state is relevant?

1. Old reason – There is a mess in light quark scalar sector. Difficult to explain observe spectrum by two quark states.
2. New reason – Some of the newly observed charmonia states may require explanation through tetraquark states.



# Two quarks VS Four quarks

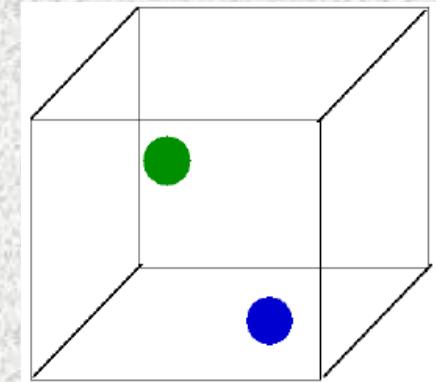


Alfred and Jaffe : Nucl.Phys.B578:367 (2000)

# Multi-particle states

A problem for finite box lattice

- ✓ Finite box : Momenta are quantized
- ✓ Lattice Hamiltonian can have both resonance and decay channel states (scattering states)



- ✓  $A \rightarrow x+y$ , Spectra of  $m_A$  and  $\sqrt{m_x^2 + p_n^2} + \sqrt{m_y^2 + p_n^2}$ ,  $p_n = \frac{2\pi n}{La}$
- ✓ One needs to separate out resonance states from scattering states

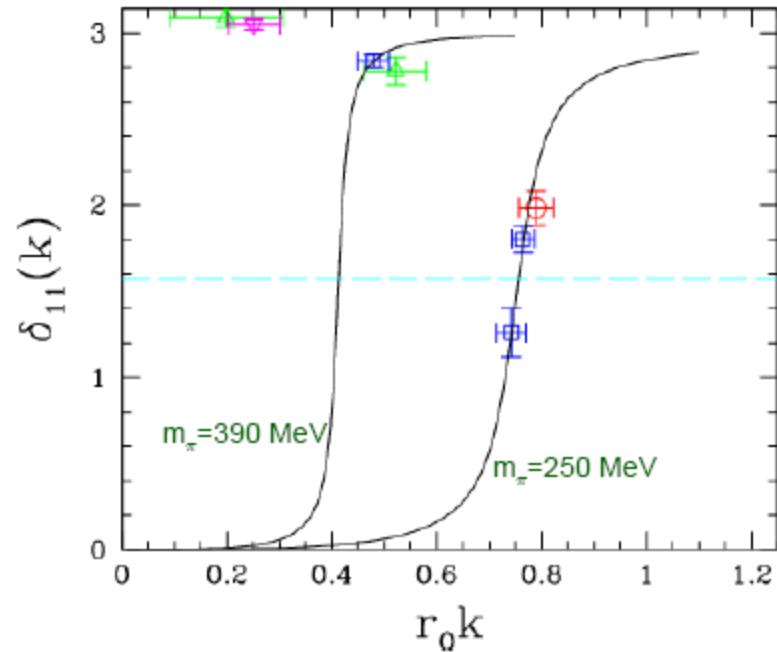
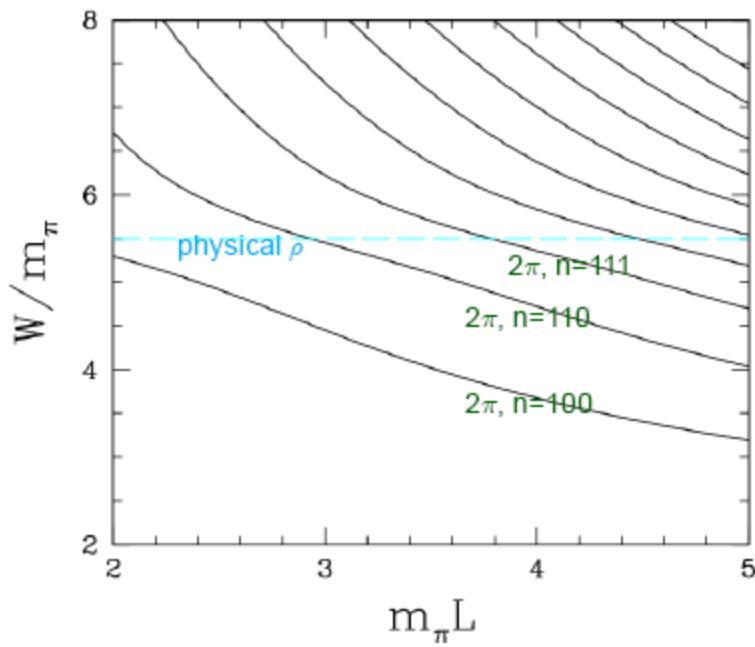
# Identifying a Resonance State

- Method 1 :
  - Study spectrum in a few volumes
  - Compare those with known multi-hadron decay channels
  - Resonance states will have no explicit volume dependence whereas scattering states will have inverse volume dependence.
- Method 2 :
  - Relate finite box energy to infinite volume phase shifts by Luscher formula
  - Calculate energy spectrum for several volumes to evaluate phase shifts for various volumes
  - Extract resonance parameters from phase shifts
  - May observe change of sign of scattering length as a function of quark mass
- Method 3 :
  - Collect energies for several volumes into momentum bin in energy histograms that leads to a probability distribution which shows peaks at resonance position ....V. Bernard et al, JHEP 0808,024 (2008)

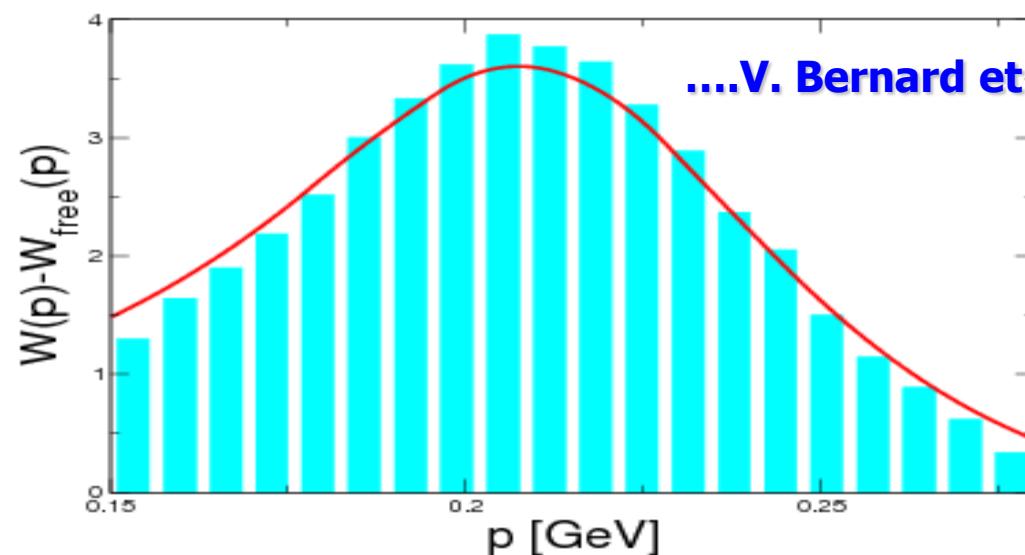
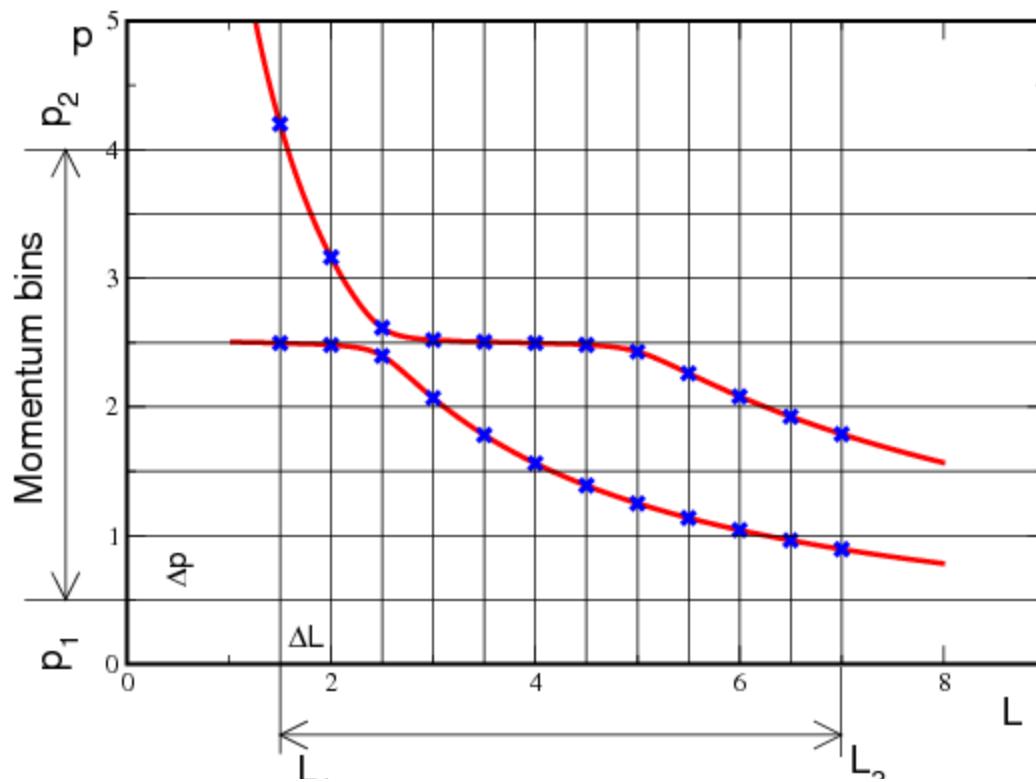
# Rho decay

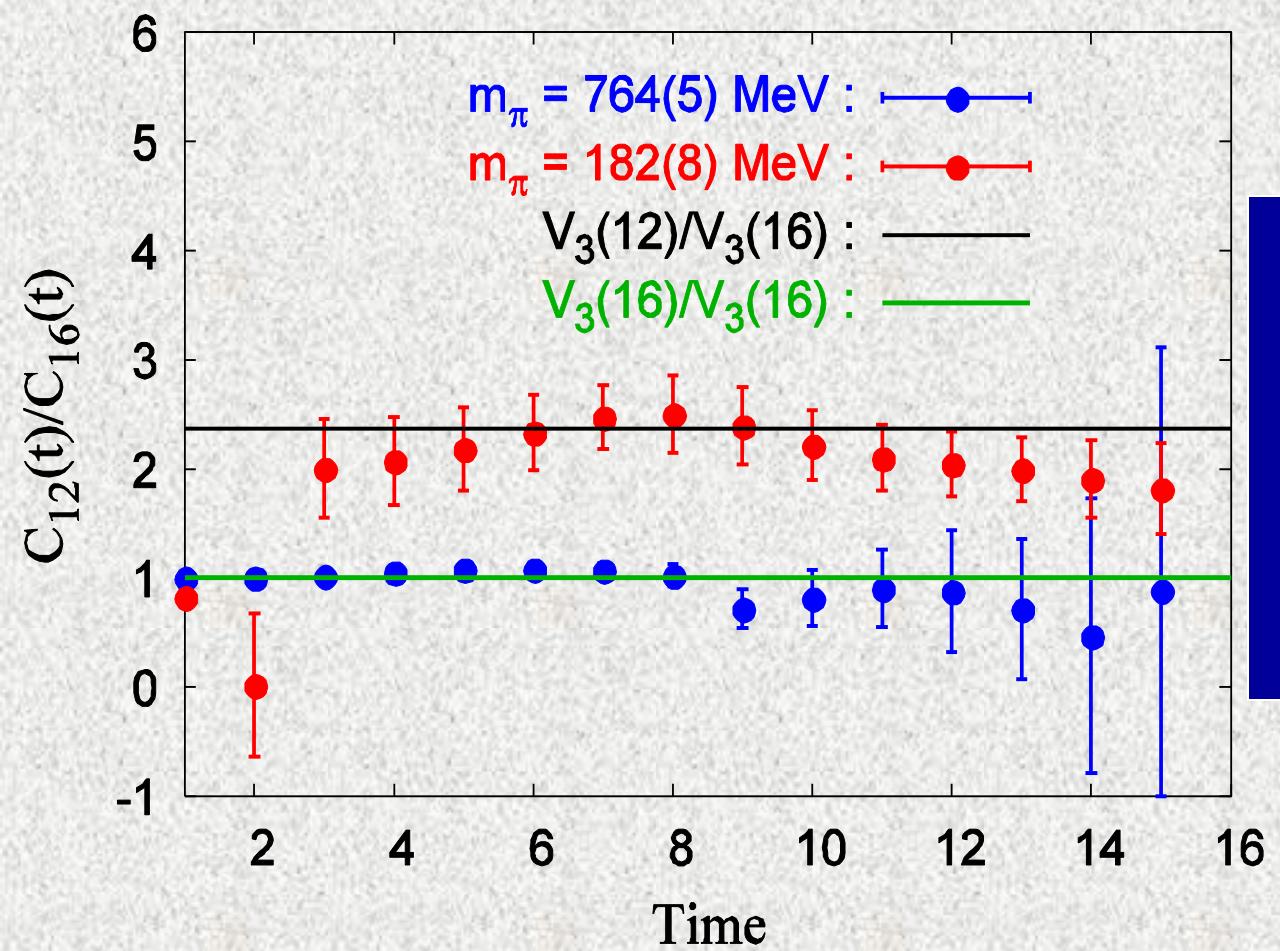
$\rho$  mass from effective range expansion: finite box

$$\frac{k^3}{W} \cot(\delta_{11}(k)) = \frac{24\pi}{g_{\rho\pi\pi}^2} (k_\rho^2 - k^2), \quad W = 2\sqrt{k^2 + m_\pi^2}, \quad k_\rho = \frac{1}{2}\sqrt{m_\rho^2 - 4m_\pi^2}$$



QCDSF 2008



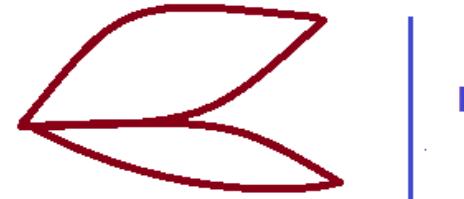
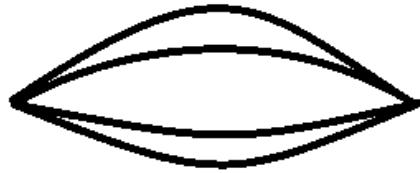
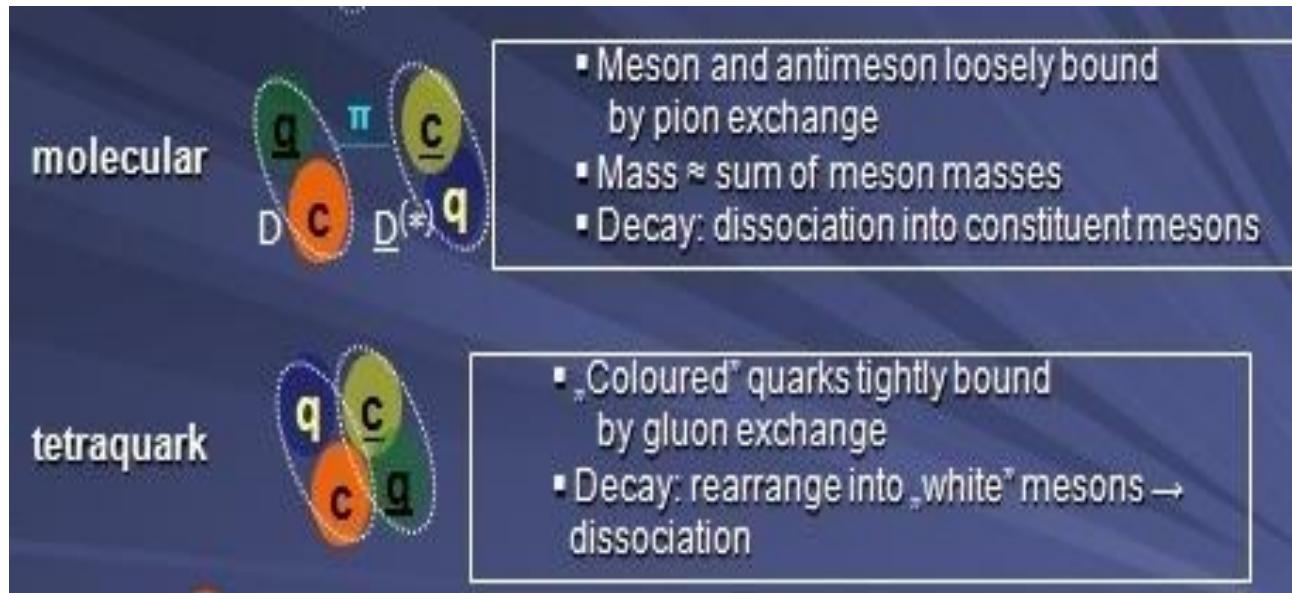


$$\frac{G_{V1}(t)}{G_{V2}(t)} = \begin{cases} \approx \frac{W_{V1}}{W_{V2}} e^{-m_0 t} & t \rightarrow \infty \\ \approx \frac{V_2}{V_1} \text{ two particle} \\ \approx 1 \text{ one particle} \end{cases}$$

**Ratio of scalar meson correlator at two volumes  
and at two different quark masses**

**Mathur,.... Liu et. al.. Phys.Rev.D76, 114505 (2007)**

# Tetraquark and Molecular States



Vary  $r$  to see if there is any structure in B-S wave function which will have implication to decaying states.

# $\sigma$ and $\kappa$

$$m_\sigma = 441_{-8}^{+16} \text{ MeV}$$

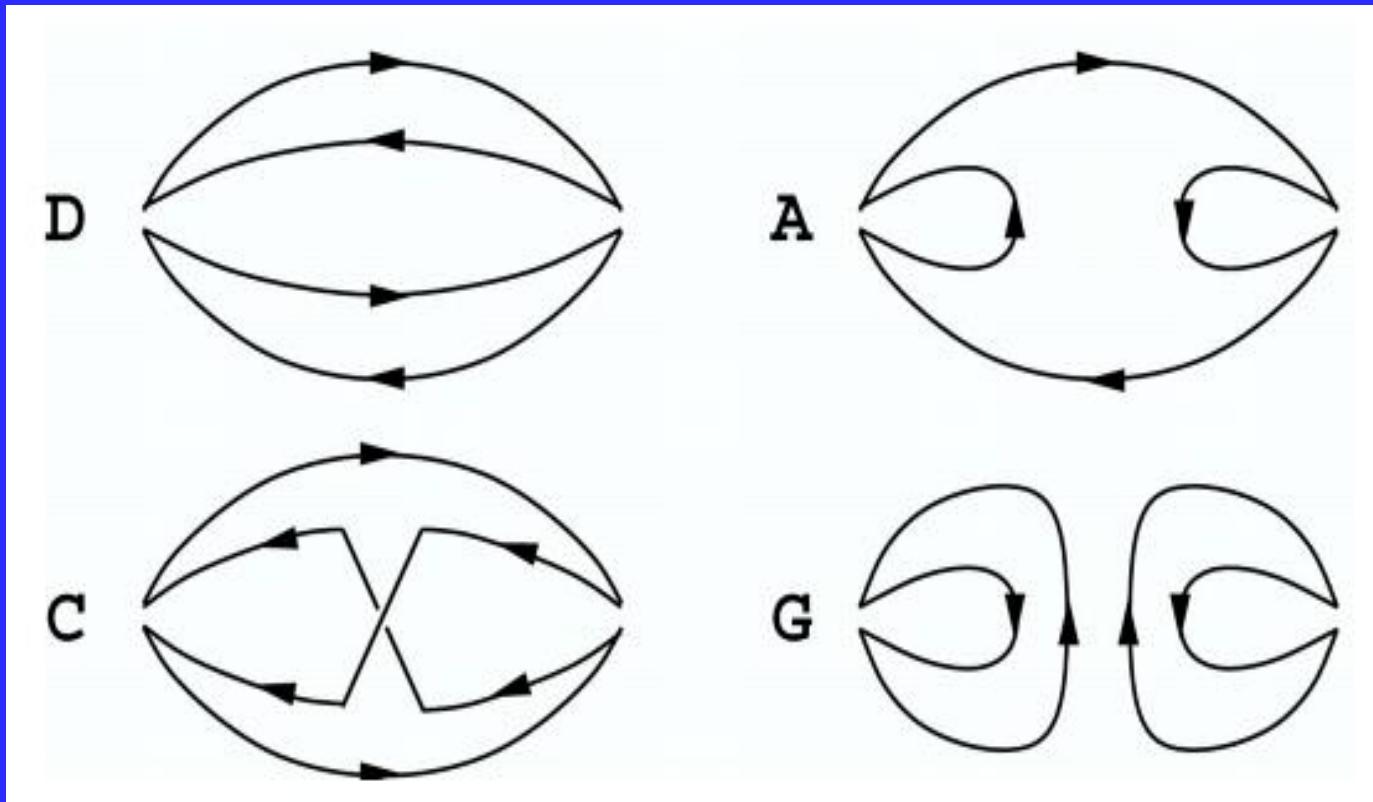
$$\Gamma_\sigma = 544_{-25}^{+18} \text{ MeV}$$

$$m_\kappa = 658 \pm 13 \text{ MeV}$$

$$\Gamma_\kappa = 557 \pm 24 \text{ MeV}$$

- I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96 (2006) 132001.
- S. Descotes-Genon and B. Moussallam, Eur. Phys. J. C48 (2006) 553.
- Note on the scalar mesons, C. Amsler et al., Review of Particle Physics, Phys. Lett. B667 (2008) 1; M. Ablikim et al., BES collaboration, Phys. Lett. B645 (2007) 19, Phys. Lett. B633 (2006) 681; G. Bonvicini et al., CLEO collaboration, Phys. Rev. D76 (2007) 012001; D. V. Bugg, arXiv:0906.3992 [hep-ph]; J.M. Link et al., FOCUS collaboration, arXiv: 0905.4846 [hep-ex]; E.M. Aitala et al., E791 collaboration, Phys. Rev. D73 (2006) 032004; I. Caprini, Phys. Rev. D77 (2008) 114019.

# Two pion state

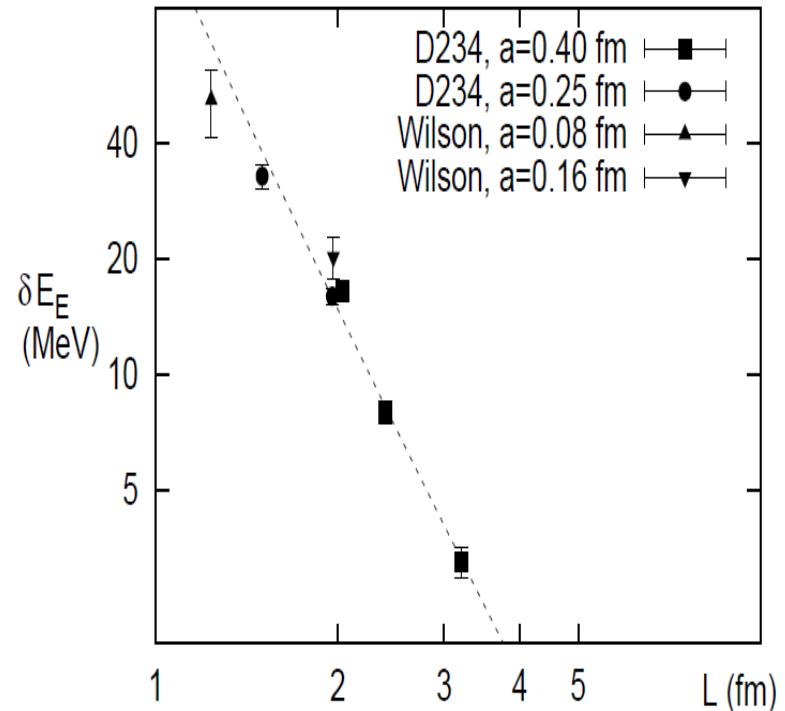
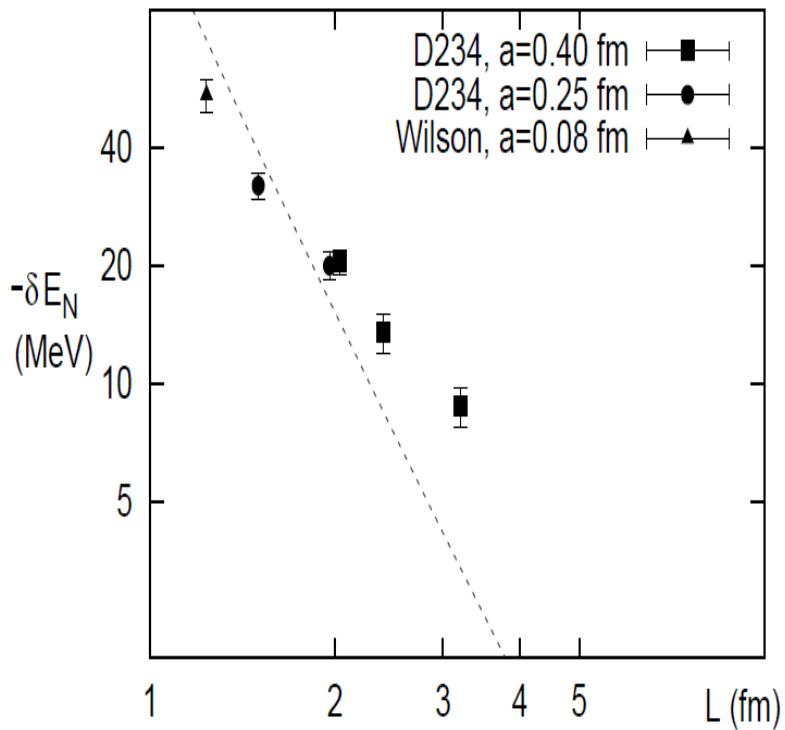


$$\left\langle \chi(t) \chi^+(0) \right\rangle = 2 \left[ D(t) + \frac{1}{2} C(t) - 3 \left( A(t) - \frac{1}{2} G(t) \right) \right], \quad I=0 ,$$

$$= D(t) - C(t) , \quad I=2$$

# Alford & Jaffe : Nucl.Phys.B578:367 (2000)

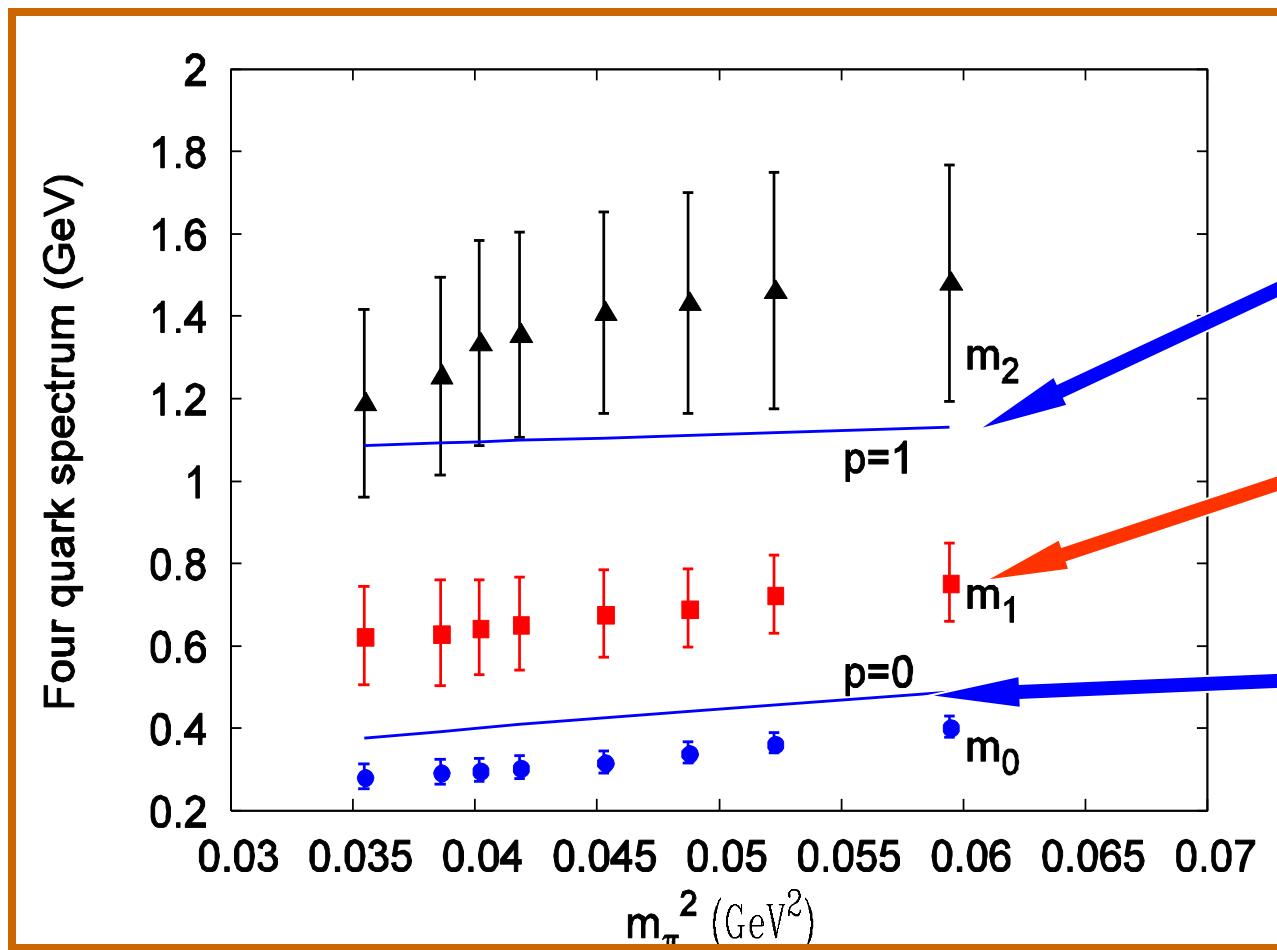
$m_\pi$  : 790 and 840 MeV



**Presence of a bound state is suggestive, but not conclusive**

$$\bar{\psi} \gamma_5 \psi \bar{\psi} \gamma_5 \psi [\pi\pi, I^G(J^{PC}) \equiv 0^+(0^{++})]$$

Mathur,.... Liu et. al.. Phys.Rev.D76, 114505 (2007)



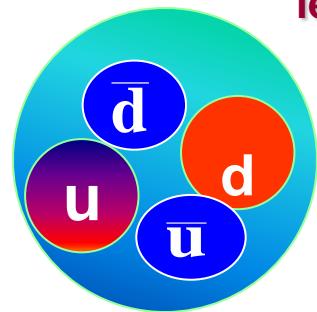
Scattering states

$$E_\pi(p=1) + E_\pi(p=1)$$

Possible BOUND state  
 $\sigma(600)?$

$$E_\pi(p=0) + E_\pi(p=0)$$

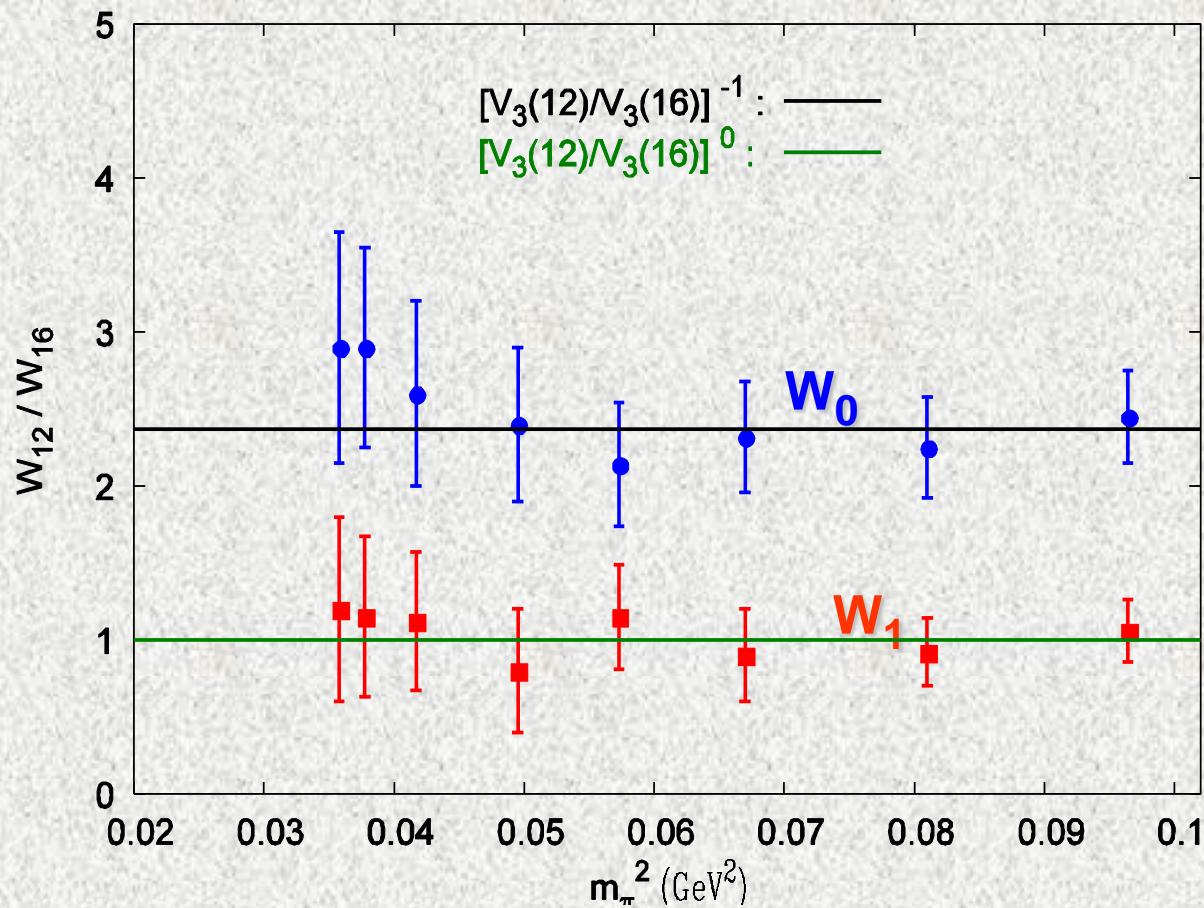
Scattering states  
(Negative scattering length)



Evidence of a tetraquark state?

# Volume dependence of spectral weights

Mathur,.... Liu et. al..Phys.Rev.D76, 114505 (2007)

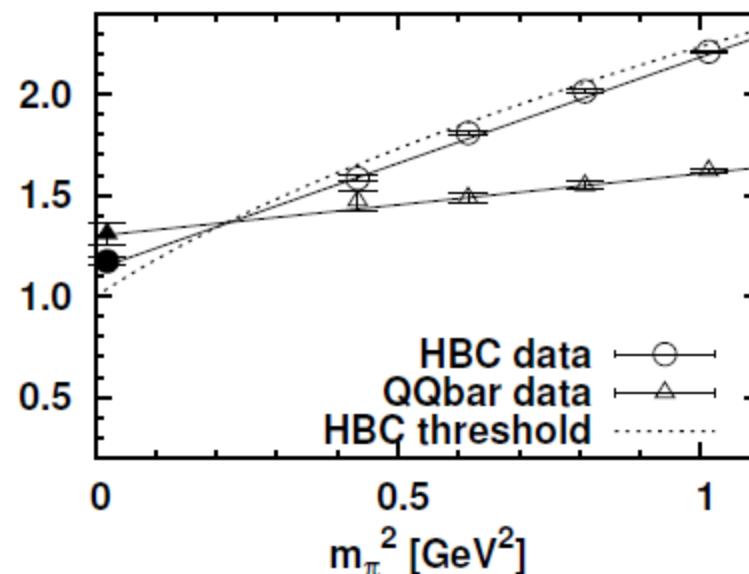
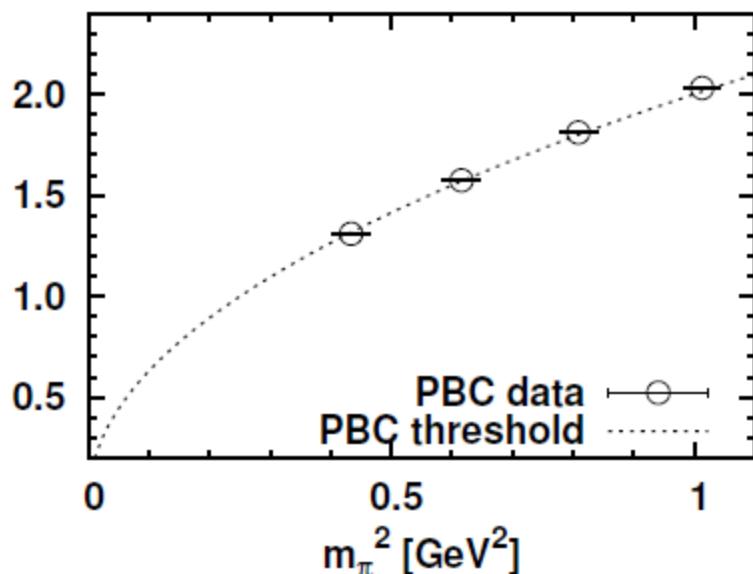


Volume independence suggests the observed state is an one particle state

- Quenched, anisotropic clover :  $a_t = 0.045 \text{ fm}$ ,  $L = 2.15 \text{ fm}$ ,  $m_s < m_q < 2m_s$
- Only ground state

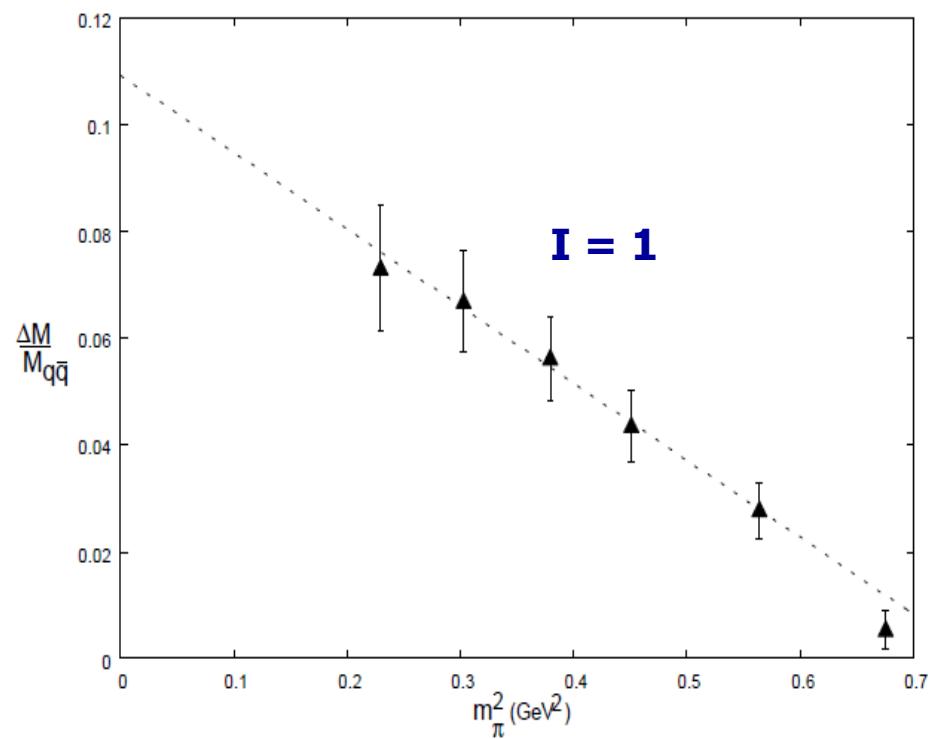
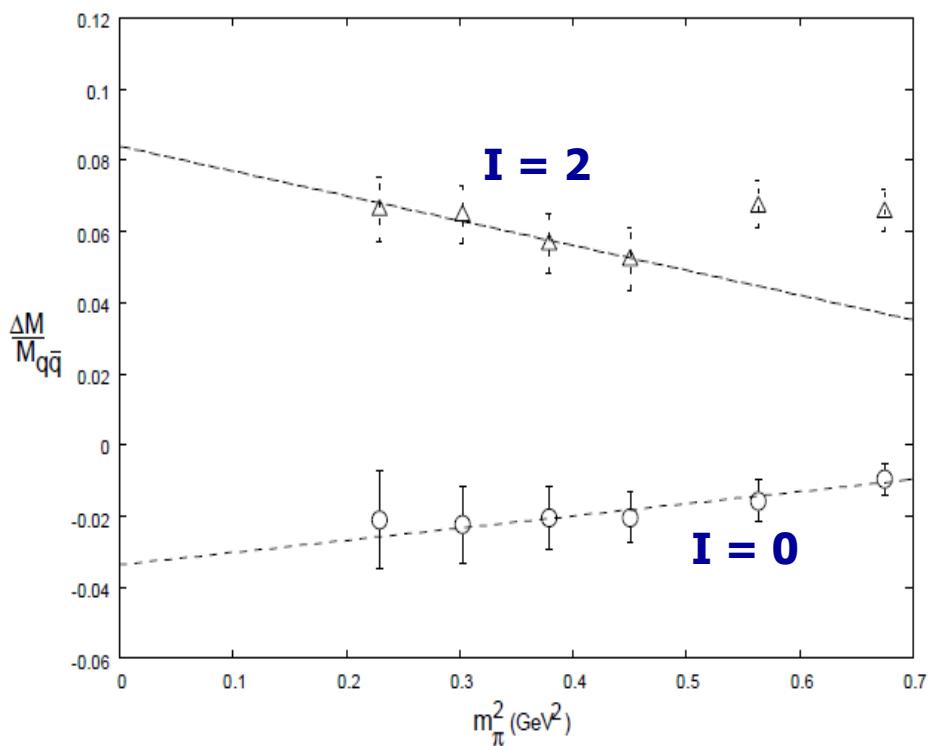
Table I. Hybrid Boundary Condition (HBC) to raise the threshold of two-meson scattering states.

|     | $u, d$        | $\bar{u}, d$ | $q\bar{q}$ -meson | two-meson threshold                            | tetra-quark ( $qq\bar{q}\bar{q}$ ) |
|-----|---------------|--------------|-------------------|--|------------------------------------|
| PBC | periodic      | periodic     | periodic          | $m_1 + m_2$                                    |                                    |
| HBC | anti-periodic | periodic     | anti-periodic     | $\sum_{k=1,2} \sqrt{m_k^2 + \vec{p}_{\min}^2}$ | periodic<br>periodic               |



No indication of tetraquark in this mass range

**Quenched, anisotropic clover fermions**



| $M_\pi (\text{GeV})$ | $\Delta M_{4q}^{I=0} / M_\rho$ | $\Delta M_{4q}^{I=2} / M_\rho$ | $\Delta M_{4q}^{I=1} / M_\rho$ |
|----------------------|--------------------------------|--------------------------------|--------------------------------|
| 0.822                | -0.0096(43)                    | 0.0660(61)                     | 0.0054(36)                     |
| 0.751                | -0.0016(54)                    | 0.0674(67)                     | 0.0278(53)                     |
| 0.672                | -0.0203(71)                    | 0.0523(86)                     | 0.0436(66)                     |
| 0.616                | -0.0206(88)                    | 0.0568 (82)                    | 0.0562(78)                     |
| 0.550                | -0.022(10)                     | 0.0650(81)                     | 0.0669(91)                     |
| 0.479                | -0.021(13)                     | 0.0664(92)                     | 0.0731(94)                     |

# Prelovsek,...et. al, arXiv : 1005.0948

Interpolating fields used :

$$\begin{aligned}
 \mathcal{O}_{i=1,2,3}^{I=0} &= \sum_{\mu=1,2,3} \frac{1}{2} (\bar{u}\Gamma_i^\mu u)(\bar{u}\Gamma_i^\mu u) + \frac{1}{2} (\bar{d}\Gamma_i^\mu d)(\bar{d}\Gamma_i^\mu d) + 2(\bar{d}\Gamma_i^\mu u)(\bar{u}\Gamma_i^\mu d) - (\bar{u}\Gamma_i^\mu u)(\bar{d}\Gamma_i^\mu d) \\
 \mathcal{O}_{i=4,5}^{I=0} &= [\bar{u}\bar{\Gamma}_i d]_a [u\Gamma_i d]_a \\
 \mathcal{O}_{i=1,2,3}^{I=1/2} &= \sum_{\mu=1,2,3} \sum_{q=u,d,s} (\bar{s}\Gamma_i^\mu q)(\bar{q}\Gamma_i^\mu u) \\
 \mathcal{O}_{i=4,5}^{I=1/2} &= [\bar{s}\bar{\Gamma}_i d]_a [u\Gamma_i d]_a . \tag{3}
 \end{aligned}$$

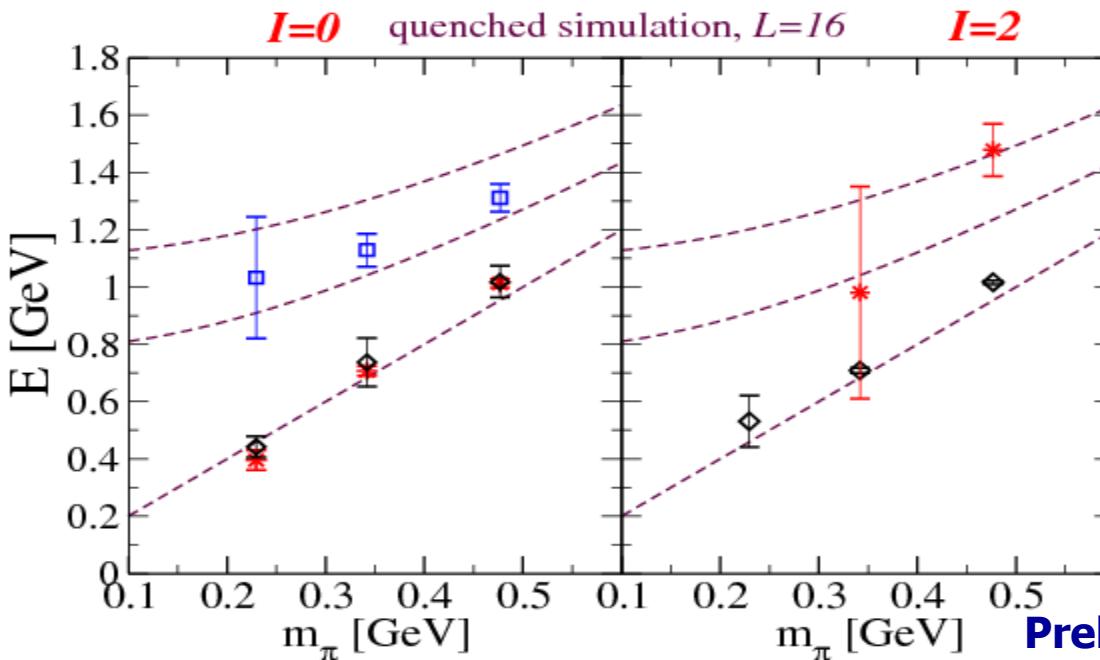
Here  $\bar{\Gamma} \equiv \gamma_0 \Gamma^\dagger \gamma_0$ ,  $[..]$  is (pseudo) scalar diquark  $[q\Gamma Q]_a \equiv \epsilon_{abc} [q_b^T \Gamma Q_c - Q_b^T \Gamma q_c]$  and

$$\Gamma_1 = \gamma_5 , \quad \Gamma_2^\mu = \gamma^\mu , \quad \Gamma_3^\mu = \gamma^\mu \gamma_5 , \quad \Gamma_4 = C \gamma_5 , \quad \Gamma_5 = C . \tag{4}$$

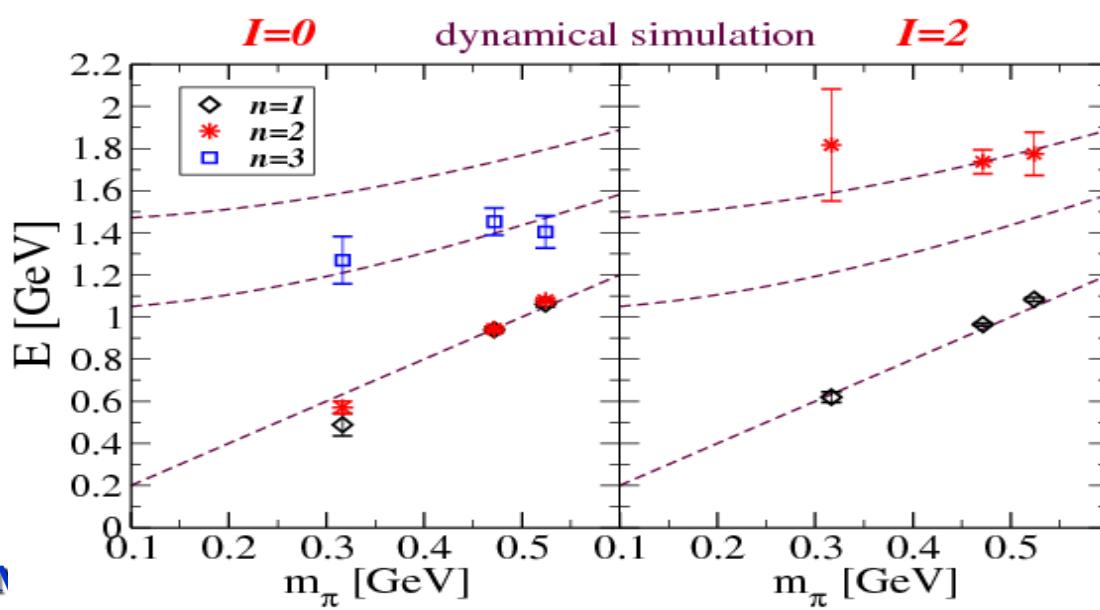
The  $I = 1/2$  tetraquark interpolators above are constructed to transform as  $(I, I_3) = (1/2, 1/2)$  under  $SU(2)_{fl}$  and like  $\bar{s}u$  flavor state under  $SU(3)_{fl}$ .

$$\begin{aligned}
 \mathcal{O}_{i=1,2,3}^{I=2} &= \sum_{\mu=1,2,3} (\bar{d}\Gamma_i^\mu u)(\bar{d}\Gamma_i^\mu u) \\
 \mathcal{O}_{i=1,2,3}^{I=3/2} &= \sum_{\mu=1,2,3} (\bar{s}\Gamma_i^\mu u)(\bar{d}\Gamma_i^\mu u)
 \end{aligned}$$

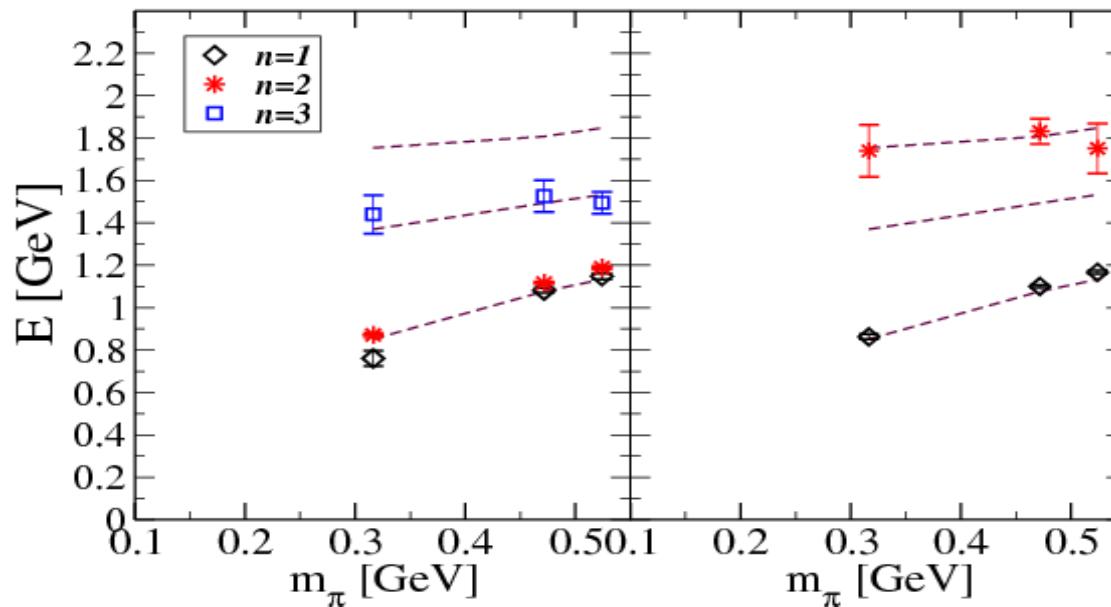
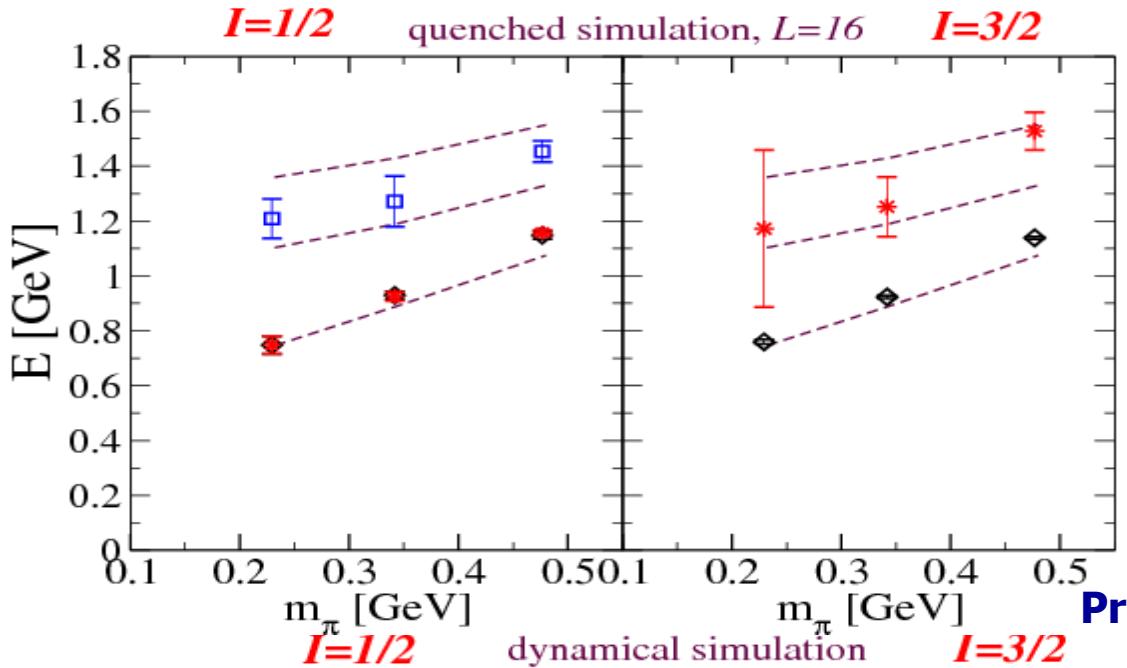
# Tetraquark States?



Prelovsek,..., NM ... et. al  
arXiv : 1005.0948

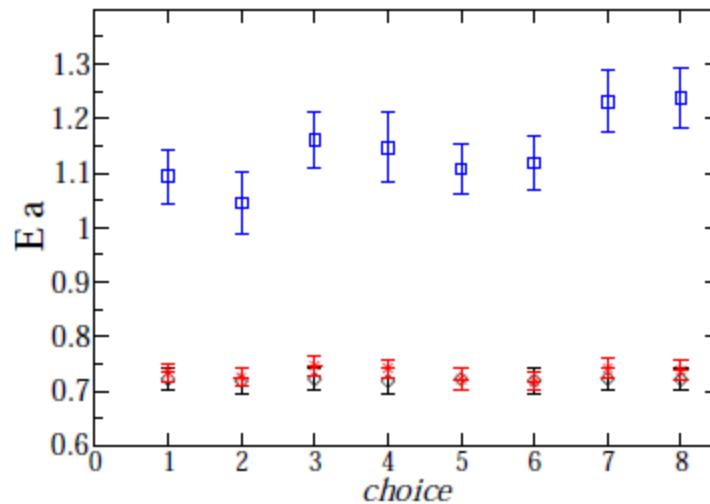


# Tetraquark States?

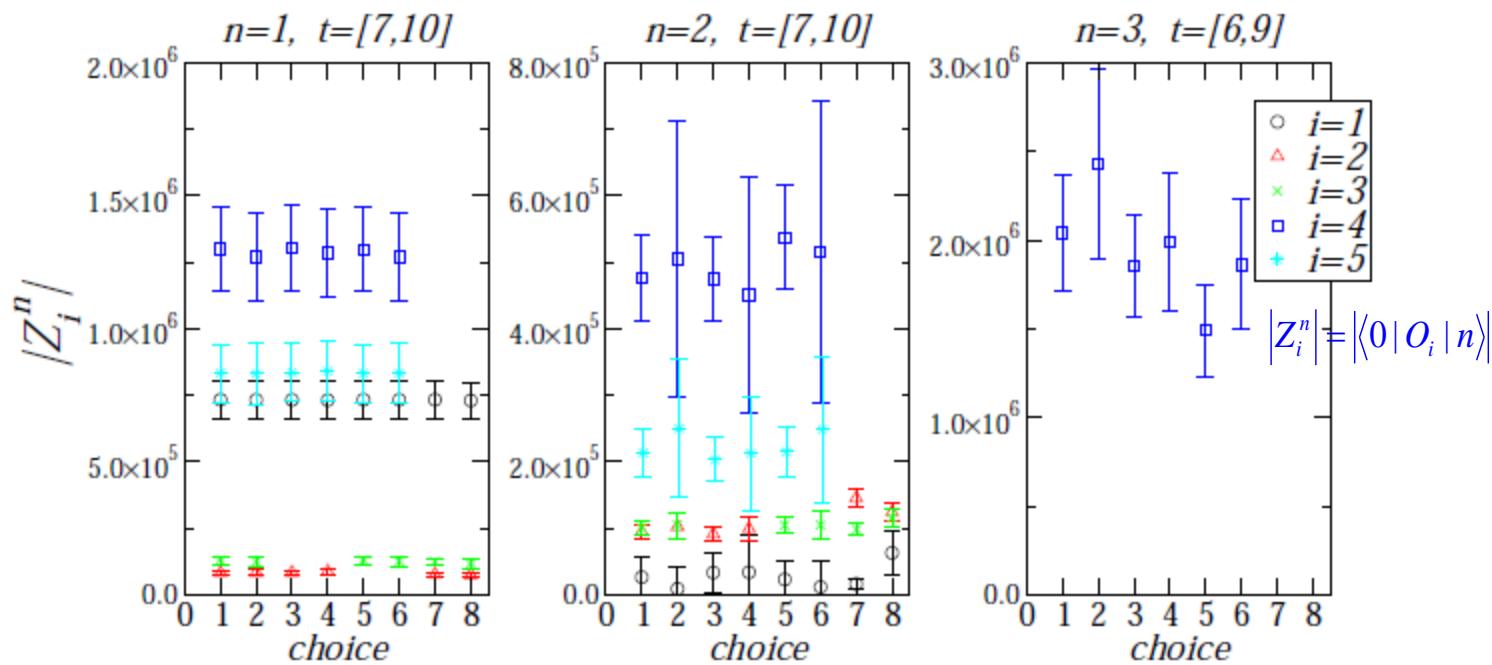


Prelovsek,..., NM ... et. al  
arXiv : 1005.0948

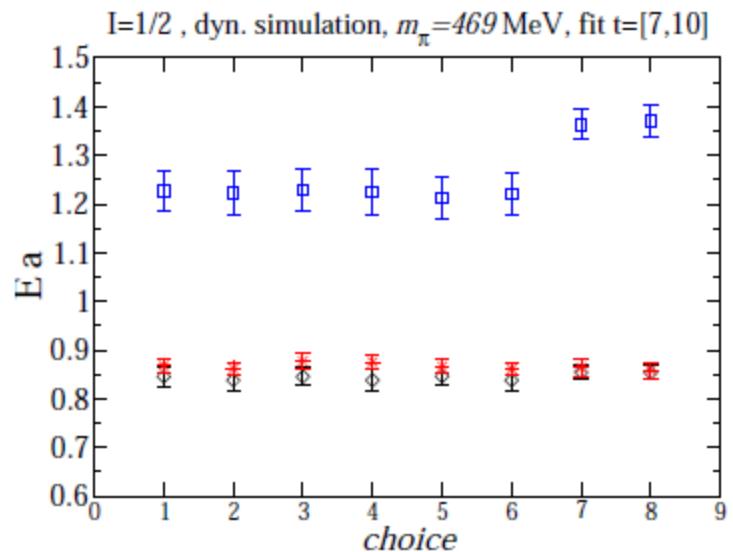
$I=0$ , dyn. simulation,  $m_\pi = 469$  MeV, fit  $t=[7,10]$



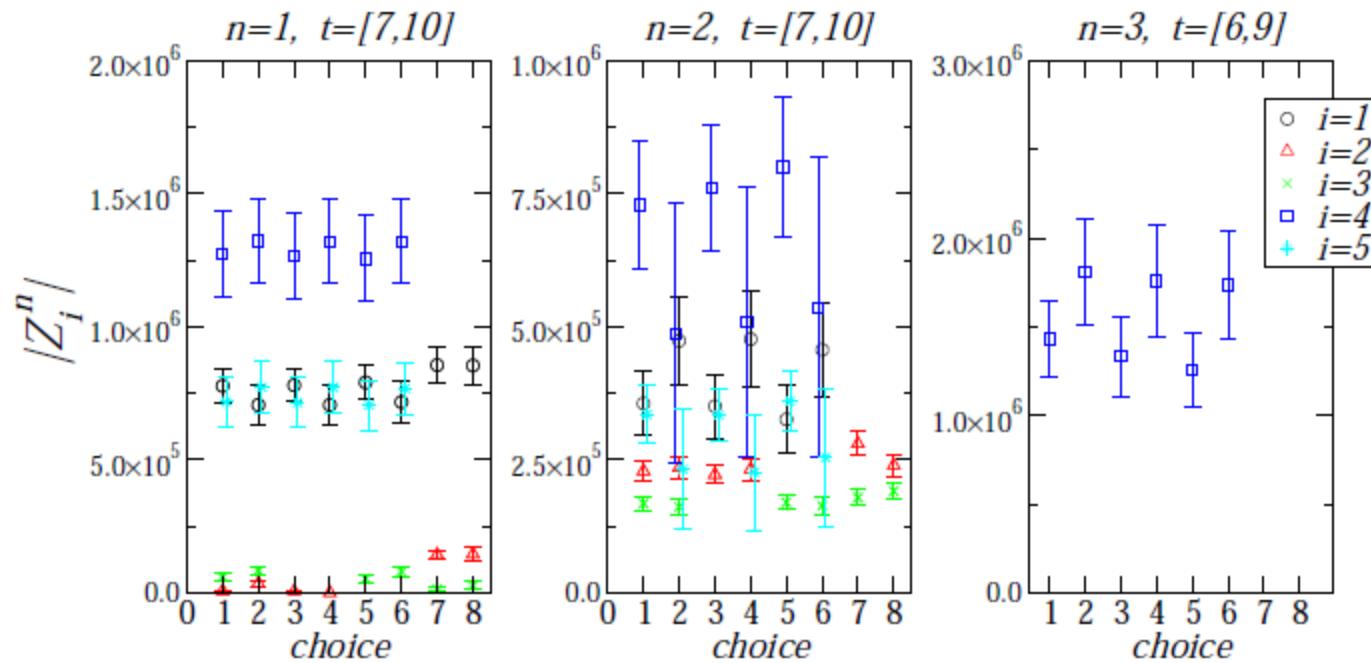
Prelovsek,..., NM ... et. al  
arXiv : 1005.0948



$$|Z_i^n| = |\langle 0 | \mathcal{O}_i | n \rangle| = \frac{|\sum_k C_{ik}(t) u_k^n(t)|}{\sqrt{\sum_{lm} |u_l^{n*}(t) C_{lm}(t) u_m^n(t)|}} e^{E_n t/2}$$



Prelovsek,..., NM ... et. al  
arXiv : 1005.0948



# New Charmonia States

| state       | $M$ (MeV)          | $\Gamma$ (MeV)     | $J^{PC}$ | Decay Modes         | Production Modes                                    | Observed by:          |
|-------------|--------------------|--------------------|----------|---------------------|---|-----------------------|
| $Y_s(2175)$ | $2175 \pm 8$       |                    |          |                     | $e^+e^-$ (ISR), $J/\psi \rightarrow \eta Y_s(2175)$ | BaBar, BESII          |
| $X(3872)$   | $3871.4 \pm 0.6$   |                    |          |                     | $B \rightarrow KX(3872)$ , $p\bar{p}$               | Belle, CDF, D0, BaBar |
| $X(3875)$   | $3875.5 \pm 1.5$   |                    |          |                     | $B \rightarrow KX(3875)$                            | Belle, BaBar          |
| $Z(3940)$   | $3929 \pm 5$       |                    |          |                     | $\gamma\gamma \rightarrow Z(3940)$                  | Belle                 |
| $X(3940)$   | $3942 \pm 9$       | $37 \pm 17$        | $J^P+$   | $DD^*$              | $e^+e^- \rightarrow J/\psi X(3940)$                 | Belle                 |
| $Y(3940)$   | $3943 \pm 17$      | $87 \pm 34$        | $J^P+$   | $\omega J/\psi$     | $B \rightarrow KY(3940)$                            | Belle, BaBar          |
| $Y(4008)$   | $4008^{+82}_{-49}$ | $226^{+97}_{-80}$  | $1^{--}$ | $\pi^+\pi^- J/\psi$ | $e^+e^-$ (ISR)                                      | Belle                 |
| $X(4160)$   | $4156 \pm 29$      | $139^{+113}_{-65}$ | $J^P+$   | $D^*\bar{D}^*$      | $e^+e^- \rightarrow J/\psi X(4160)$                 | Belle                 |
| $Y(4260)$   | $4264 \pm 12$      | $83 \pm 22$        | $1^+$    |                     | $+e^-$ (ISR)  | BaBar, CLEO, Belle    |
| $Y(4350)$   | $4361 \pm 13$      | $74 \pm 18$        | $1^+$    |                     | $+e^-$ (ISR)  | BaBar, Belle          |
| $Z(4430)$   | $4433 \pm 5$       | $45^{+35}_{-18}$   | $1^+$    |                     | $\rightarrow KZ^\pm(4430)$                          | Belle                 |
| $Y(4660)$   | $4664 \pm 12$      | $48 \pm 15$        | $1^+$    |                     | $+e^-$ (ISR)  | Belle                 |
| $Y_b$       | $\sim 10,870$      | ?                  | $1^+$    |                     | $+e^-$ (ISR)  | Belle                 |

S. Olsen arXiv:0801.1153v1 (hep-ex)

# Z<sup>+</sup>(4430)

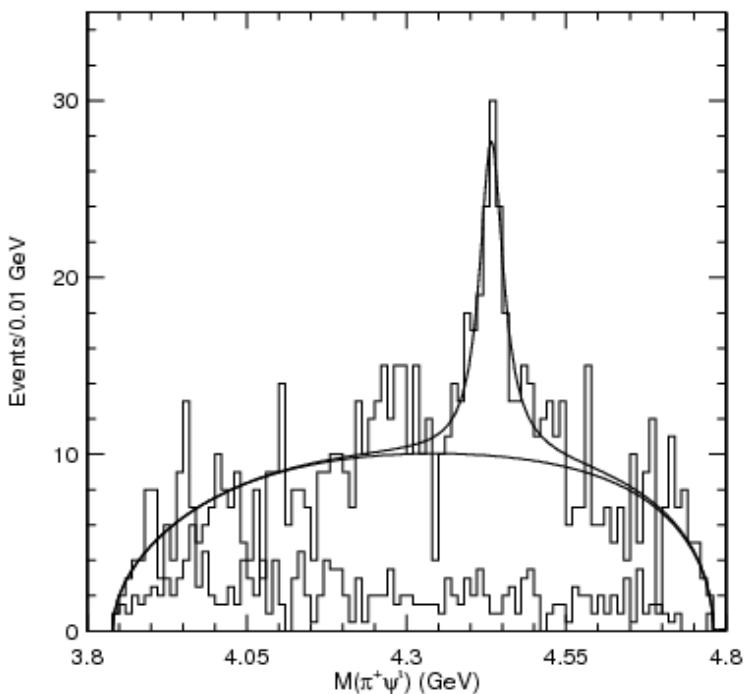
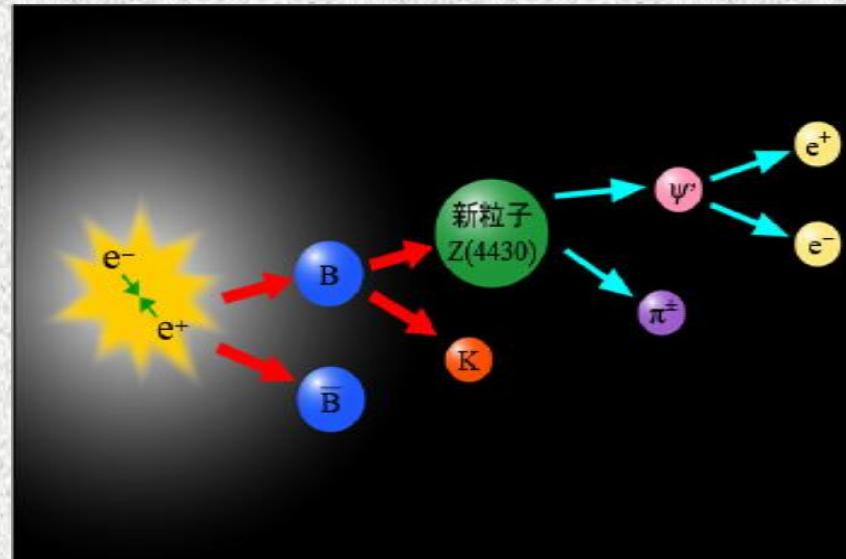


Fig. 7. The  $M(\pi^\pm \psi')$  distribution for  $B \rightarrow K\pi^\pm \psi'$  decays (from Belle.)



**Belle,**  
**S-K. Choi et al, PRL 100, 142001 (2008)**

$$M = (4433 \pm 4 \pm 2) \text{ MeV}$$

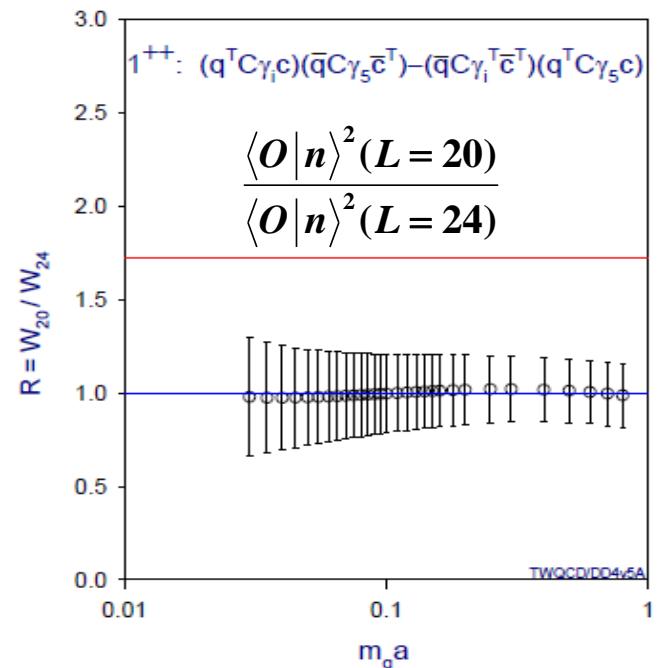
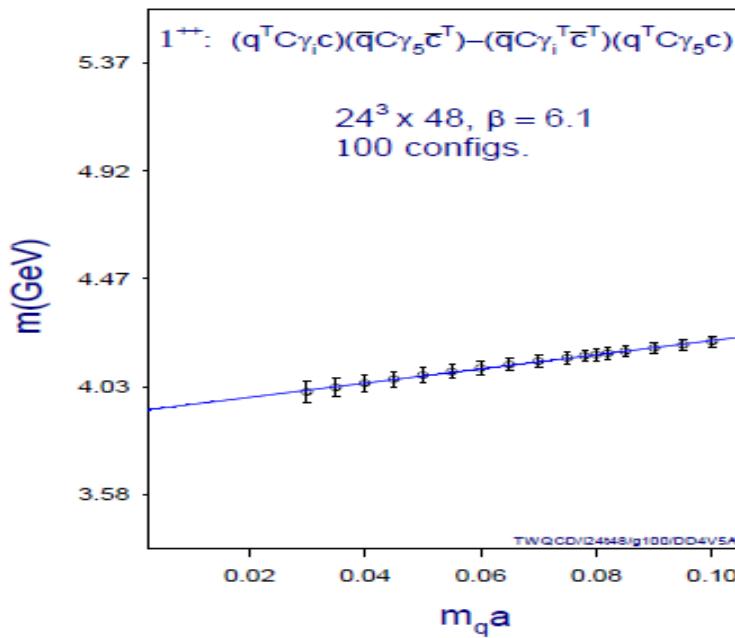
$$\Gamma = (45^{+18}_{-13}(\text{stat})^{+30}_{-13}(\text{syst})) \text{ MeV}$$

Since charged, it cannot be either charmonium or hybrid  
**Recent Babar data do not support it**  
**However, more recent Belle analysis still claims it**  
 So tetraquark?  
 $(c\bar{u})(c\bar{d})$   
 Small Charm state inside a large light quark hadron?  
 Study in baryon is needed

$$D^* D_1^{**}$$

**TWQCD** : Chiu & Hsieh, PLB646 (2006) 95, PRD73 (2006) 111503

- Quenched, overlap fermions for both light and charm
  - $m_\pi > 430 \text{ MeV}$ ,  $0.4m_s < m_q < m_c$ ,  $a = 0.09 \text{ fm}$ ,  $L=1.8 \text{ fm}, 2.2 \text{ fm}$
  - Extracted only ground state



$$\bar{c} \bar{u} c u: \ m = 3890 \pm 30 \text{ MeV} \quad X(3872) \quad DD^*, \quad m(D) + m(D^*) = 3874 \text{ MeV}$$

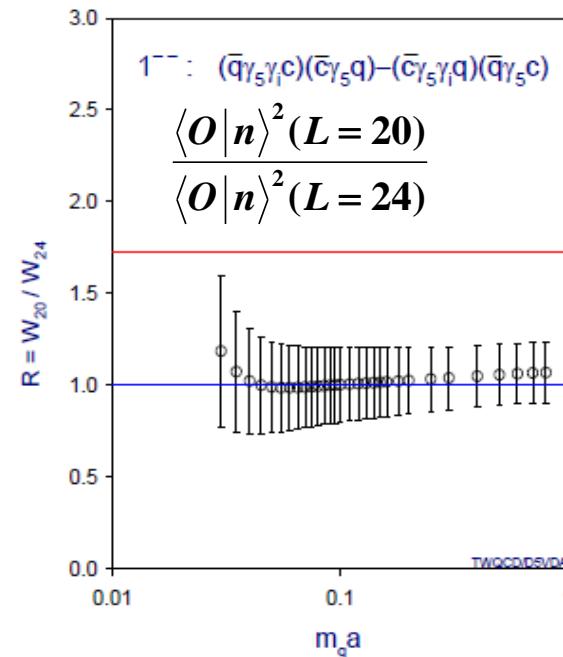
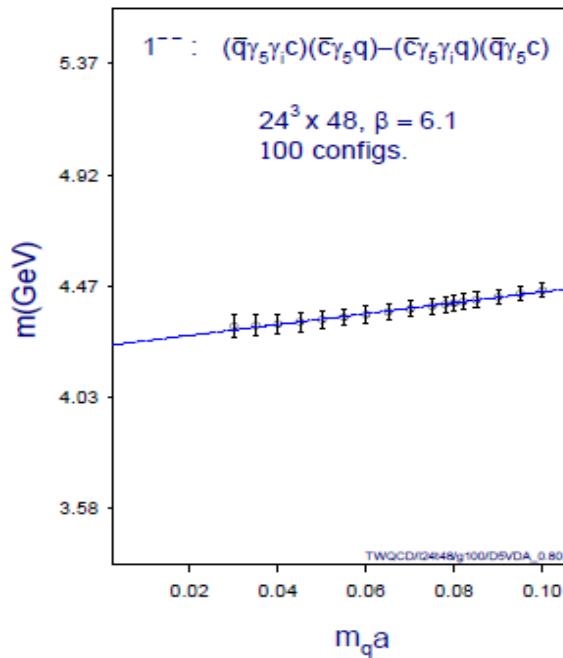
$$\bar{c}u\bar{c}s: \quad m = 4010 \pm 50 \text{ MeV} \qquad \qquad D^* \phi, \quad m(D^*) + m(\phi) = 3980 \text{ MeV}$$

**c̄sc̄s:**  $m = 4100 \pm 50 \text{ MeV}$   $Y(4140)$   $J/\psi\phi$ ,  $m(J/\psi) + m(\phi) = 4120 \text{ MeV}$

$\bar{c}c\bar{c}c$ :  $m \equiv 6080 \pm 30$  MeV

# TWQCD : Chiu & Hsieh, PRD73 (2006) 094510

- Quenched, overlap fermions for both light and charm
- $m_\pi > 430 \text{ MeV}$ ,  $0.4m_s < m_q < m_c$ ,  $a = 0.09 \text{ fm}$ ,  $L=1.8 \text{ fm}, 2.2 \text{ fm}$
- Extracted only ground state



$\bar{c}\bar{u}cu : m = 4238 \pm 31 \text{ MeV } Y(4260)$

$\bar{c}\bar{s}cs : m = 4450 \pm 100 \text{ MeV}$

$\bar{c}\bar{c}cc : m = 6400 \pm 50 \text{ MeV}$

Need to calculate scattering states before conclusion

# Liuming Liu : PoS(lat09)099

- Staggered sea
- Only ground state

$$O = (\bar{c} \gamma_5 q)(\bar{q} \gamma_i c) = DD^*$$

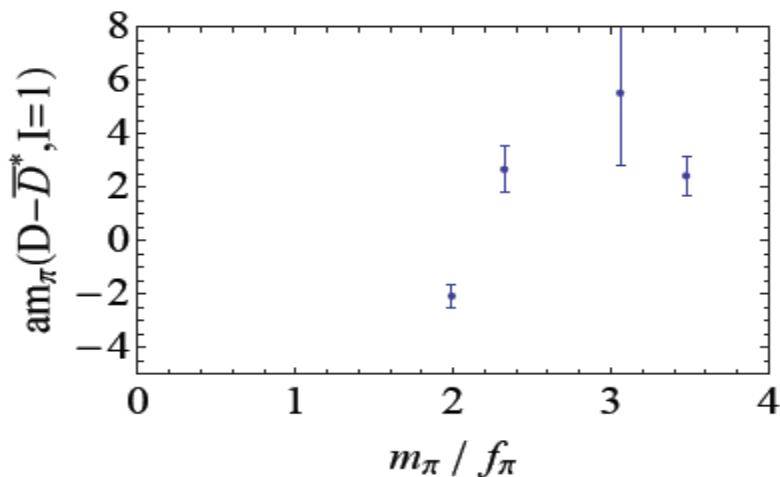
$$I = 1, \quad J^{PC} = 1^{++}$$

$$\mathcal{O}_{\eta_c - \pi}(t) = \eta_c(t)\pi^+(t) \quad \mathcal{O}_{J/\Psi - \pi}(t) = J/\Psi(t)\pi^+(t) \quad \mathcal{O}_{\eta_c - N}(t) = \eta_c(t)N(t)$$

$$\mathcal{O}_{J/\Psi - N} = J/\Psi(t)N(t) \quad \mathcal{O}_{D_s - \pi} = D_s\pi^+ \quad \mathcal{O}_{D - \pi}^{I=1} = D^+\pi^+$$

$$\mathcal{O}_{D - \bar{K}}^{I=1} = D^+\bar{K}^0 \quad \mathcal{O}_{D - \bar{K}}^{I=0} = D^+K^- - D^0\bar{K}^0 \quad \mathcal{O}_{D - K}^{I=1} = D^+K^+$$

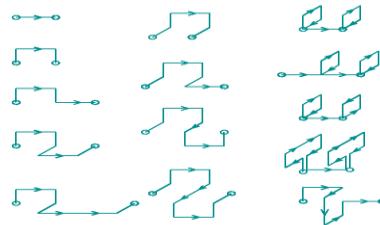
$$\mathcal{O}_{D_s - K} = D_s^+K^+ \quad \mathcal{O}_{D - \bar{D}^*}^{I=1} = D^+\bar{D}^{0*}$$



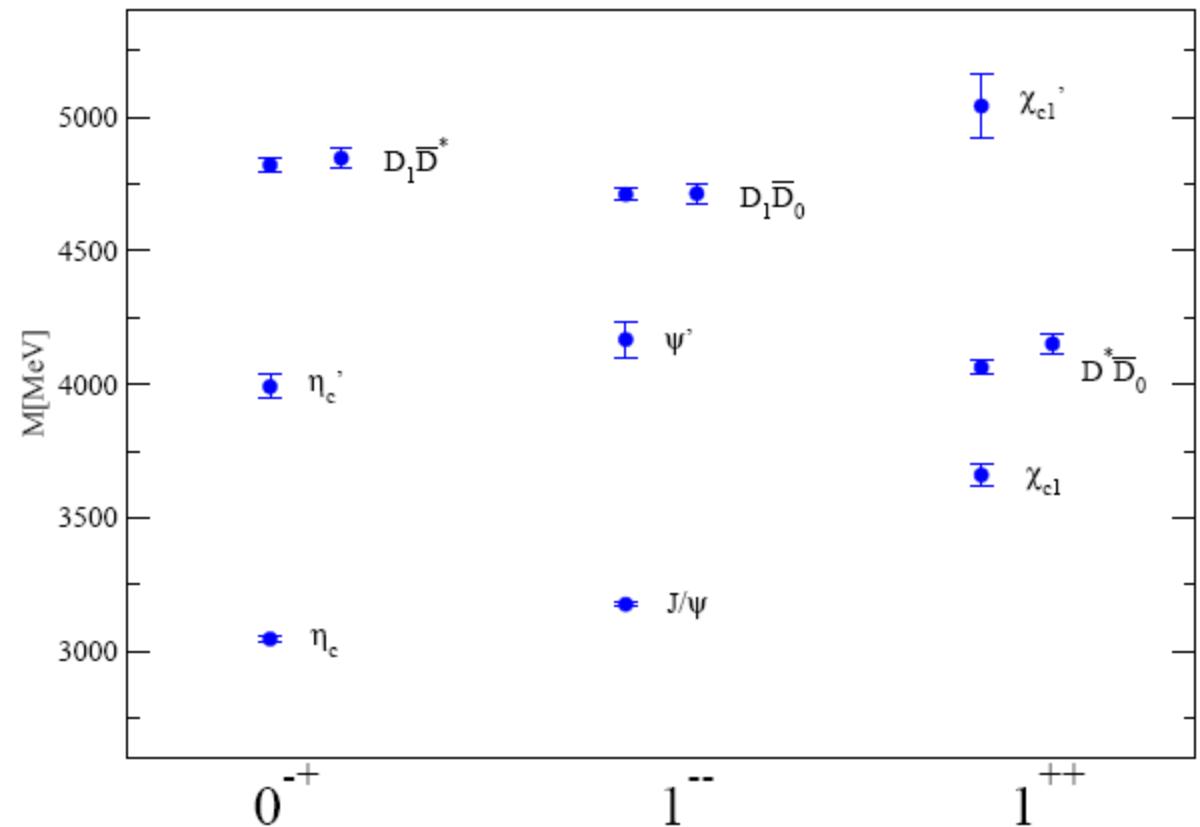
Change of sign of scattering length indicates possible bound state?

# How to distinguish various states?

- Local :  $\bar{q}(x)\Gamma q(x)$
- Tetraquark :  $\bar{q}(x)\Gamma_1 q(x)\bar{q}(x)\Gamma_2 q(x)$
- Molecular :  $\bar{q}(x)\Gamma_1 q(x)\bar{q}(y)\Gamma_2 q(y)$
- Hybrid :  $\bar{q}(x)\Gamma D q(x)$
- Glueball :



- Solve generalized eigenvalue problem including all operators and find contribution from each state



| state          | $(cc\bar{c})_l$ | $(cc\bar{c})_n$ | $(c\bar{c}u\bar{u})_l$ | $(c\bar{c}u\bar{u})_n$ |
|----------------|-----------------|-----------------|------------------------|------------------------|
| $\eta_c$       | 0.54(3)         | -0.02(1)        | -0.1(1)                | -0.31(5)               |
| $D_1\bar{D}^*$ | 0.07(1)         | 0.01(1)         | -0.46(8)               | 0.14(2)                |
| $J/\psi$       | 0.51(4)         | -0.03(1)        | 0.09(1)                | 0.21(6)                |
| $D_1\bar{D}$   | 0.08(6)         | 0.04(1)         | -0.18(1)               | 0.53(4)                |
| $\chi_{c1}$    | 0.39(5)         | 0.69(3)         | -0.22(3)               | -0.49(4)               |
| $D\bar{D}^*$   | 0.63(4)         | -0.23(3)        | -0.73(4)               | 0.12(3)                |

$$|\eta_c\rangle = \frac{1}{\mathcal{N}} \left( |cc\bar{c}\rangle + \lambda \frac{\langle c\bar{q}q\bar{c}|H_1|cc\bar{c}\rangle}{E(cc\bar{c}) - E(c\bar{q}q\bar{c})} |c\bar{q}q\bar{c}\rangle \right)$$

**G. Bali : arXiv:0911.1238**

# PANDA

*Antiproton Annihilation at Darmstadt at the High Energy Storage Ring at GSI*

*An Experiment at FAIR "Facility of Antiproton and Ion Research"*

Special detector, high luminosity ( $L \sim 2 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ ) and phase space cooled antiproton beam.

Energy resolution  $\sim 50 \text{ keV}$

Physics :

- Charonium spectroscopy
- Excited Glue  
(Glueballs and Hybrids)
- Charm in Nuclei
- Charmonium Hypernuclei
- D and  $D_s$ -Physics
- Other Topics

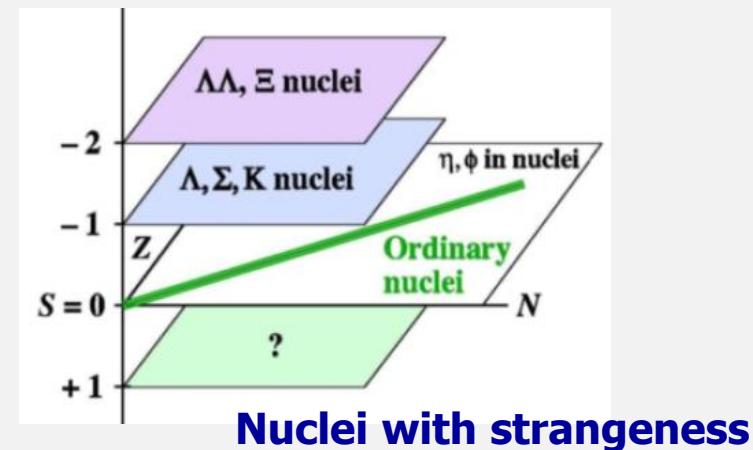
<http://www-panda.gsi.de>





# Physics

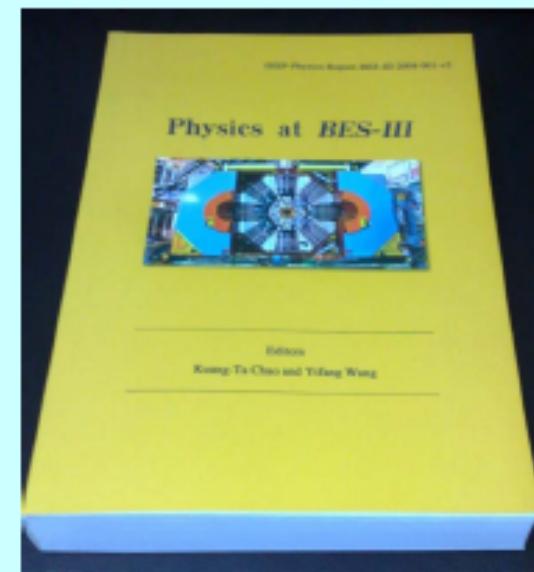
- ✓ Strangeness nuclear physics, hypernuclei, kaonic nuclei



- ✓ Exotic hadron search, Chiral dynamics and meson properties in nuclear medium
- ✓ Structure function, hard exclusive processes, spin structure of the nucleon with target polarization
- ✓ Hadron physics in neutrino scattering

# Physics at BEPCII/BESIII

- Precision measurement of CKM matrix elements
- Precision test of Standard Model
- Light hadron spectroscopy
- Charmonium physics
- Search for new physics/new particles



hep-ex/0809.1869

| Physics Channel | Energy (GeV) | Luminosity ( $10^{33} \text{ cm}^{-2}\text{s}^{-1}$ ) | Events/year          |
|-----------------|--------------|---|----------------------|
| J/ $\psi$       | 3.097        | 0.6   | $1.0 \times 10^{10}$ |
| $\tau$          | 3.67         | 1.0   | $1.2 \times 10^7$    |
| $\psi'$         | 3.686        | 1.0   | $3.0 \times 10^9$    |
| D*              | 3.77         | 1.0   | $2.5 \times 10^7$    |
| Ds              | 4.03         | 0.6   | $1.0 \times 10^6$    |
| Ds              | 4.14         | 0.6   | $2.0 \times 10^6$    |

# Conclusion

- + There are several lattice calculations with some sort of indications about the presence of fourquark states in light quark domain but no conclusive evidence yet. One needs to use dynamical calculations with disconnected insertion diagrams.
- + In heavy quark region several experimental states could be candidates for fourquark states. There are no comprehensive lattice calculation yet.